Flashing ratchet model with high efficiency

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As a simple model of the Brownian motor, we consider hopping motion of a particle in a periodic asymmetric double-well potential which randomly switches between two states. The potential profiles of the states are identical but shifted by half a period. The current and the efficiency are explicitly calculated as functions of the parameters of the model, including also a load force. Such a flashing ratchet is shown to be particularly efficient, with the efficiency tending to unity when the highest peak of the potential is high enough to suppress the backward motion.

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I. INTRODUCTION

Nonequilibrium fluctuations combined with broken reflection symmetry (usually provided by a periodic but asymmetric potential) can cause directed motion even in the absence of any macroscopic bias force. Model systems for nonequilibrium transport based on the rectification of Brownian motion, which are generically called *ratchets* or *Brownian motors*, have received much attention in recent years and been discussed in manifold contexts by approaches of varying rigor and sophistication (see, e.g., Refs. [1–3] for a comprehensive review). The main motivation comes from biological applications relevant to ion pumps [4] and motors (myosin on filament track (actin), which plays a key role in muscular contraction [5], kinesin motors in eucaryotic cells on microtubules [6]) and also from particle segregation experiments [7]. Two main types of Brownian motors are recognized: *flashing* ratchets where transported particles are exposed to a time-fluctuating energy profile [8–10] and *rocking* ratchets where the particles experience the action of a fluctuating force [11].

In description of the model, of foremost interest is usually the *velocity* (in the long-time limit) or the stationary *current* of the particles. Considerable recent attention has been focused on another important quantity, the energetic *efficiency*, with which a Brownian motor converts fluctuations into useful work, e.g., advances against an external force (see Ref. [12] for a general review). Some of actual molecular motors exhibit a very high efficiency of energy conversion [13] while flashing ratchets which have attractive features for biological systems and particle separation are often recognized as systems with inefficient energy transduction. Such an opinion is based on the results of detailed analyses of so-called *fluctuating potential* models [8], in particular the *on-off* ratchet scheme [9]. For these models, diffusive steps are necessary for directed motion to occur, which evidently leads to substantial energy losses, and that is why efficiency no better than a few percents has been reported (see Refs. [1,12] and references therein). On the other hand, Chauwin, Ajdari, and Prost [10] have shown that if the potential switches between the states with identical spatial periods and with extrema appropriately shifted relative to each other, Brownian motion is not involved in the transport generation. Numerical analysis of the model with mutually shifted potentials [14] has demonstrated that its efficiency is more than order of magnitude larger than for the models requiring diffusion. Thus, with a special setup, not only rocked [15], but also flashing ratchets are able to convert energy into directed motion with high efficiency.

In the present work, we exploit the idea of mutually shifted potentials invoking a simple hopping model in which a combination of thermally activated barrier surmounting with the switching between two shifted potential profiles leads to directed motion. Our main result is that the flashing ratchet efficiency can be rather high despite the fact that Brownian motion is involved as a necessary element of the operating cycle. Moreover, it is shown that the efficiency tends to the ideal limit of unity when the barriers preventing the backward motion become high enough to suppress it completely.

II. THE MODEL

Consider a particle (motor) moving in a potential energy profile $U(x)$ that describes the interaction between the particle and a track (filament) as a function of the position of the particle along the track. The potential profile is represented by a periodic sequence of wells and barriers (see Fig. 1).

For the sake of simplicity, we assume that the periodic unit of the length $2L$ consists of two wells with the energies $\pm u$ (from here on the energy is expressed in units of $k_BT=1$, where k_B is the Boltzmann number and T is the absolute temperature). The centers of the wells are located at $L/2$ and $3L/2$. The positions of the potential peaks, V and v, are 0 and L, respectively. Note that within each unit, the reflection symmetry is broken.

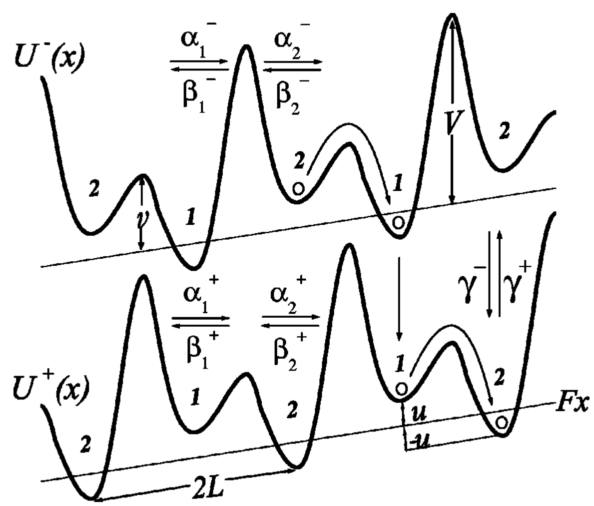

FIG. 1. Schematic representation of the model for a Brownian motor, with the parameters of the potential profiles, the load-induced linear potential, and the transition rates indicated (see text for details). To exemplify, a particle initially found in a well 2 in the state $-$ is expected to move into well 1 to the right, if the lifetime of the state $-$ is large enough. After the potential has flipped to the state $+$, the particle presumably escapes to well 2 thus shifting to the right again. So, the combination of thermally activated barrier surmounting with the potential modulation leads to the uphill flow of particles.

We also assume that the potential can take two forms, $U^+(x)$ and $U^-(x)$, which are identical copies of each other, randomly shifted on half a period to the right or to the left, $U^-(x)=U^+(x\pm L)$. The transitions between the $+$ and the $-$ states may be caused by an external signal or by a far-from-equilibrium chemical reaction resulting in a conformational change of the particle or of the track. They occur at the moments of time that are chosen from an exponential waiting time distribution. In other words, the switching dynamics is considered as a stationary Markov process symbolized by the rate equation

$$+ \underset{\gamma^-}{\overset{\gamma^+}{\rightleftharpoons}} -$$

According to this scheme, the average time spent by the particle in the $+(-)$ state is $(\gamma^{+(-)})^{-1}$.

The model is represented in Fig. 1 where the parameters of the potential profiles are introduced and the linear potential arising from the load force $-F$ $(F>0)$ is added. In this model, the potential fluctuations which provide the energy input and a detailed balance violation are combined with structural asymmetry [16]. Thus, the model has all elements required to operate as a Brownian motor.

There are two ways to treat transport in this model. The first is based on the Fokker-Plank description in which the joint probability density for the particle to be in state $+$ or $-$ at position x at time t, $\mathcal{P}^{\pm}(x,t)$, satisfies two Fokker-Plank equations with source terms

$$\frac{\partial \mathcal{P}^+(x,t)}{\partial t} = -\frac{\partial J^+(x,t)}{\partial x} - \gamma^+\mathcal{P}^+(x,t) + \gamma^-\mathcal{P}^-(x,t),$$

$$\frac{\partial \mathcal{P}^-(x,t)}{\partial t} = -\frac{\partial J^-(x,t)}{\partial x} + \gamma^+\mathcal{P}^+(x,t) - \gamma^-\mathcal{P}^-(x,t), \tag{1}$$

while the currents $J^{\pm}(x,t)$ resulting from interaction with the track, the action of the load force, and diffusion are given by

$$J^{\pm}(x,t) = -\frac{D}{k_BT}\left[\frac{\partial U^{\pm}(x)}{\partial x} + F\right]\mathcal{P}^{\pm}(x,t) - D\frac{\partial \mathcal{P}^{\pm}(x,t)}{\partial x}, \tag{2}$$

where D is the diffusion coefficient taken equal in both states. The distributions $\mathcal{P}^{\pm}(x,t)$ must also satisfy the periodic boundary conditions

$$\mathcal{P}^{\pm}(x+2L,t) = \mathcal{P}^{\pm}(x,t), \quad J^{\pm}(x+2L,t) = J^{\pm}(x,t), \tag{3}$$

and the normalization condition

$$\int_0^{2L}[\mathcal{P}^+(x,t)+\mathcal{P}^-(x,t)]dx = 1. \tag{4}$$

This approach is rigorous for the entire range of the potential modulation frequency. However, explicit analytical solutions can only be obtained in special cases of potential profiles.

With an alternative approach, the particle motion is considered as hopping between wells (discrete states) due to thermal activation. Such a kinetic formulation is more convenient in view of the purposes of the present study, since it allows simple analytical expressions to be obtained for the quantities of interest, which thus become easy to analyze. However, the kinetic description is valid when the external modulation of the potential is slow compared with the intrawell relaxation frequency k_0. Another assumption is that the potential barriers between any two wells are larger than k_BT.

Applying the kinetic approach to the model sketched in Fig. 1, note that with periodic boundary conditions imposed on the system, it is sufficient to consider only one period. Let $\rho_i^{\pm}(t)$ be the probability of finding the particle in the ith ($i=1,2$) well in the state $+$ or $-$ at time t. Evidently,

$$\rho_1^+(t)+\rho_2^+(t)+\rho_1^-(t)+\rho_2^-(t) = 1 \tag{5}$$

for all values of t. As Fig. 1 suggests, the time evolution of the occupancy probabilities is governed by a master equation of the form

$$\frac{d}{dt}\begin{pmatrix}\rho_1^+\\ \rho_2^+\\ \rho_1^-\\ \rho_2^-\end{pmatrix}=\begin{pmatrix}-\alpha_1^+-\beta_1^+-\gamma^+ & \alpha_2^++\beta_2^+ & \gamma^- & 0\\ \alpha_1^++\beta_1^+ & -\alpha_2^+-\beta_2^+-\gamma^+ & 0 & \gamma^-\\ \gamma^+ & 0 & -\alpha_1^--\beta_1^--\gamma^- & \alpha_2^-+\beta_2^-\\ 0 & \gamma^+ & \alpha_1^-+\beta_1^- & -\alpha_2^--\beta_2^--\gamma^-\end{pmatrix}\begin{pmatrix}\rho_1^+\\ \rho_2^+\\ \rho_1^-\\ \rho_2^-\end{pmatrix}. \tag{6}$$

The α's and the β's are the transition rates (to the right and to the left) defined by the Arrhenius-like expressions [18]:

$$\alpha_1^+=\alpha_2^-=k_0\exp(-v+u-f),$$

$$\alpha_2^+=\alpha_1^-=k_0\exp(-V-u-f),$$

$$\beta_1^+=\beta_2^-=k_0\exp(-V+u+f),$$

$$\beta_2^+=\beta_1^-=k_0\exp(-v-u+f), \tag{7}$$

where $f=FL/(2k_BT)$. Note that $\alpha_1^\pm\alpha_2^\pm=\beta_1^\pm\beta_2^\pm$ for $f=0$, in accordance with the condition of the detailed balance, which is valid within each state but violated for the whole system due to switching between states.

It should be mentioned that various versions of the discrete model for Brownian motors have been proposed and analyzed (see Refs. [1,2]). At first glance, the model proposed here closely resembles those introduced by Ambaye and Kehr [20] and by Astumian [21]. However, our model is different from the toy model [20] in that we assume the potential profiles of states to be identical (but shifted) copies whereas the model [20] consists of two states, one with asymmetric and the other with symmetric hopping rates. A comparison of our model with that in Ref. [21] reveals two main distinctions: (i) we consider the $+\rightarrow-$ and $-\rightarrow+$ transitions both driven externally, whereas Astumian assumed the second transition to occur spontaneously; (ii) in Ref. [21], the heights of the barriers controlling the backward motion are the same for wells 1 and 2 (both in the $+$ and $-$ states), i.e., all the β's are equal, whereas we suppose that the left barrier can be higher than the right one, i.e., $\beta_1^+=\beta_2^-<\beta_2^+=\beta_1^-$. Recall that the models introduced in Refs. [20] and [21] exhibit a low efficiency. By contrast, the present model allows us to construct flashing ratchet with a drastically higher energy transduction efficiency, as demonstrated in the following section. It is notable that the high efficiency of energy conversion can also be reached when the energy of the alternating electric field which interacts with charged transporters is used to pump uncharged ligands across the membrane against a concentration gradient [22,23] [a phenomenon known as "electroconformational coupling" (ECC) [4]]. There exists a formal analogy between the present model and the ECC model, which will be discussed more fully elsewhere.

III. RESULTS AND DISCUSSION

Master equation (6) determines the particle motion. After transient effects have died out, the system approaches a steady state characterized by $d\rho_i^\pm/dt=0$. The steady-state solution of Eq. (6) satisfying the normalization condition, Eq. (5), can be written as

$$\rho_{1,2}^+=\frac{\gamma^-}{\Gamma(\Gamma+\Sigma)}\left[\alpha_{2,1}^++\beta_{2,1}^++\gamma^\pm\mp\frac{(\alpha_2^++\beta_2^+)(\gamma^+-\gamma^-)}{\Sigma}\right],$$

$$\rho_{1,2}^-=\rho_{2,1}^++\frac{(\alpha_{1,2}^++\beta_{1,2}^+)(\gamma^+-\gamma^-)}{\Gamma\Sigma}. \tag{8}$$

[using double sign notation ($\pm$ or $\mp$), we imply that the upper sign refers to the first subscript and the lower sign refers to the second subscript]. Here we have introduced the overall rates $\Sigma=\alpha_1^++\beta_1^++\alpha_2^++\beta_2^+$ and $\Gamma=\gamma^++\gamma^-$ characterizing the frequency of hopping between the wells and of the potential fluctuations, respectively. Additionally, in the steady state, Eq. (1), when integrated over x, takes the form

$$J^\pm(x)\pm\int_0^x[\gamma^+\mathcal{P}^+(x')-\gamma^-\mathcal{P}^-(x')]dx'=J^\pm(0). \tag{9}$$

In what follows, we use Eq. (9) and the steady-state distribution specified by Eq. (8) to discuss the directed current of particles and the energetic efficiency which are the quantities of foremost interest in the context of Brownian motor operation.

A. Current

A stationary solution implies the constant position-independent current $J=J^+(x)+J^-(x)$. Within the kinetic approach, the current is simply the hopping rate to the right minus the hopping rate to the left for wells 1 and 2 both in the $+$ and $-$ states (see Fig. 1):

$$J=\alpha_1^+\rho_1^++\alpha_1^-\rho_1^--\beta_2^+\rho_2^+-\beta_2^-\rho_2^-. \tag{10}$$

Using Eq. (8), the current is expressible in the physically suggestive form

$$J=\frac{\alpha_1^+\alpha_2^+-\beta_1^+\beta_2^+}{\Sigma}+\gamma^*\frac{(\alpha_1^+-\alpha_2^+)^2-(\beta_1^+-\beta_2^+)^2}{\Sigma(\Gamma+\Sigma)}, \tag{11}$$

where $(\gamma^*)^{-1}=(\gamma^+)^{-1}+(\gamma^-)^{-1}$. The first term in Eq. (11) independent of the potential modulation frequency represents a negative (downhill) current due to the load force F and disappears in the unloaded ($F=0$) regime. The second term plays a key role in describing the ratchet effect. It predicts the positive (uphill) particle transport subject to the threefold condition: (i) flips between two potential states (potential flashes) do occur (both γ^+ and γ^- are nonzero); (ii) the spatial asymmetry is provided ($\alpha_1^+ \neq \alpha_2^+$ or $\beta_1^+ \neq \beta_2^+$); (iii) the load force F is not too large. Note that at the fixed flip rate Γ, the quantity γ^* (and hence the uphill current) takes its maximal value when $\gamma^+ = \gamma^- \equiv \gamma$. Such a symmetric driving is thus the most advantageous one and will be referred to as *optimal* modulation.

To more fully examine how the current as a function of the applied force behaves upon variation of the model parameters, it is expedient to rewrite the current, Eq. (11), with the rates defined in Eq. (7). For the optimal modulation, the corresponding expression is given by

$$J(f)/k_0 = \frac{1}{2}\,\frac{e^{-v}\sinh(u-f) - e^{-V}[\sinh(u+f) + 2\zeta\sinh(2f)]}{1+\zeta[\cosh(u-f) + e^{-V+v}\cosh(u+f)]}, \tag{12}$$

where $\zeta \equiv (k_0/\gamma)e^{-v}$.

First, we focus on the unloaded situation, since this case is usually of special interest. For $F=0$, Eq. (12) is reduced to

$$J(0)/k_0 = \frac{1}{2}\, e^{-v}\sinh(u)\,\frac{1-e^{-V+v}}{1+\zeta\cosh(u)(1+e^{-V+v})}. \tag{13}$$

One readily sees that $J(0)$ is zero for $\gamma = 0$ and linearly increases with γ in the slow modulation regime, $\gamma \ll k_0 e^{-v}\cosh u$. As the flip rate is raised further, Eq. (13) predicts a monotonic increase in $J(0)$ with γ and the current saturation at infinite flip rate. This is, however, a formerly noticed [20] artifact of the kinetic approach which is invalid at fast flip rates, $\gamma \gg k_0$. What actually happens in the fast modulation regime is that the current diminishes with rising γ (and tends to zero when $\gamma \to \infty$) because the time between switches becomes too short to establish an equilibrium distribution between wells 1 and 2 and the mechanism of directed motion thus breaks down. This is the case for any type of the flashing ratchet model, as pointed out in Ref. [1].

The uphill current is a monotonically decreasing function of f. An important characteristic of a Brownian motor is a value of the force f_s (usually named stopping force), at which an exact cancellation of the ratchet effect takes place, i.e., $J(f_s) = 0$. Equation (11) suggests two mechanisms for the behavior of the function $J(f)$ and the value f_s: (i) an increase (in modulus) of the negative current due to the load, as specified by the first term in Eq. (11); (ii) equalization of potential wells 1 and 2 with the enhancement of the load force [the relative well energy is $2(u-f)$ in the presence of an external field (see Fig. 1)], which evidently diminishes the second term responsible for the ratchet effect. Note that the first term vanishes when the highest barrier tends to infinity. For such impenetrable barriers, it is the second mechanism that solely causes the current to decrease with f and the stopping force takes its maximal value $f_s = u$. In this case ($V \gg 1$), Eq. (12) takes an especially simple form when the potential modulation is slow, $\zeta \gg 1$:

$$J(f) = \frac{1}{2}\gamma\tanh(u-f). \tag{14}$$

This result is easily obtained also in terms of the Fokker-Plank approach. Indeed, using Eq. (9) and relation $J^{\pm}(x) = J^{\mp}(x+L)$ which follows from the condition $U^-(x) = U^+(x+L)$, the current can be expressed as

$$J = J^+(0) + J^-(0) = \gamma R(L) + 2J^+(0), \tag{15}$$

where $R(x) = \int_0^x [\mathcal{P}^-(x') - \mathcal{P}^+(x')]dx'$. Provided a potential fluctuates very slowly, the distribution fully relaxes to either $U^+(x)$ or $U^-(x)$ after every flip, i.e., the probability densities $\mathcal{P}^{\pm}(x)$ in the low-frequency domain are mainly determined by the potential profiles $U^{\pm}(x)$, whereas the role of the parameters γ ($\gamma^+ = \gamma^- = \gamma$ in the optimal modulation regime considered here) and D is insignificant. Let $x=0$ corresponds to the position of the highest peak of U^+; then $J^+(0)$ represents the backward motion which is negligible when $V \gg 1$. In estimating $R(L)$ note that when the barriers are high, the main contribution into the integral comes from the vicinity of the potential well located at $L/2$. The potential profiles are assumed identical in this region, $U^+(x) \simeq U^-(x) + 2u$, and hence we have $\mathcal{P}^-(x) \simeq \exp[2(u-f)]\mathcal{P}^+(x)$ for x close to $L/2$. From these relations and the normalization condition, Eq. (4), it follows that $R(L) \simeq (1/2)\tanh(u-f)$. Thus in the case $\zeta, V \gg 1$, the current calculated in terms of the Fokker-Plank equation, Eq. (9), coincides with the result of the kinetic approach, Eq. (14), as one would expect.

The typical behavior of the steady-state current as a function of the applied force is exemplified in Fig. 2 for different values of the potential parameters. As is seen from panel (a), both the current and the stopping force increase with V. Also, note that the effect of V on the current is negligible at small values of the load and becomes significant as the load grows. Such a behavior is caused not only by the reduction of the load-induced negative current due to the rise in V [see the first term in Eq. (11)] but also by the suppression of the backward motion which accordingly increases the second term in Eq. (11). At large values of V, the current takes the largest value and the stopping force approaches its limit $f_s = u$. More precisely, the large-V asymptotic behavior of the stopping force is given by

$$f_s \simeq [1-(1/2+\zeta)\varepsilon]u, \tag{16}$$

where $\varepsilon \equiv 2e^{-V+v}\sinh(2u)/u$ is small, $\varepsilon \ll 1$.

The relative well energy $2u$ (along with the potential modulation) plays a role of "a driving force" for the directed particle transport. It is not surprising, then, that a rise in u serves to increase the current and the stopping force [see panel (b)]. The influence of the parameter v on J and f_s is just opposite to that of V [see panel (c)], since an increase in v leads the positive current to reduce [see the second term in Eq. (11)].

In our model, the velocity $\mathcal{U}$ of unidirectional motion induced by the flips between the potential states is related to the steady-state current by the equation

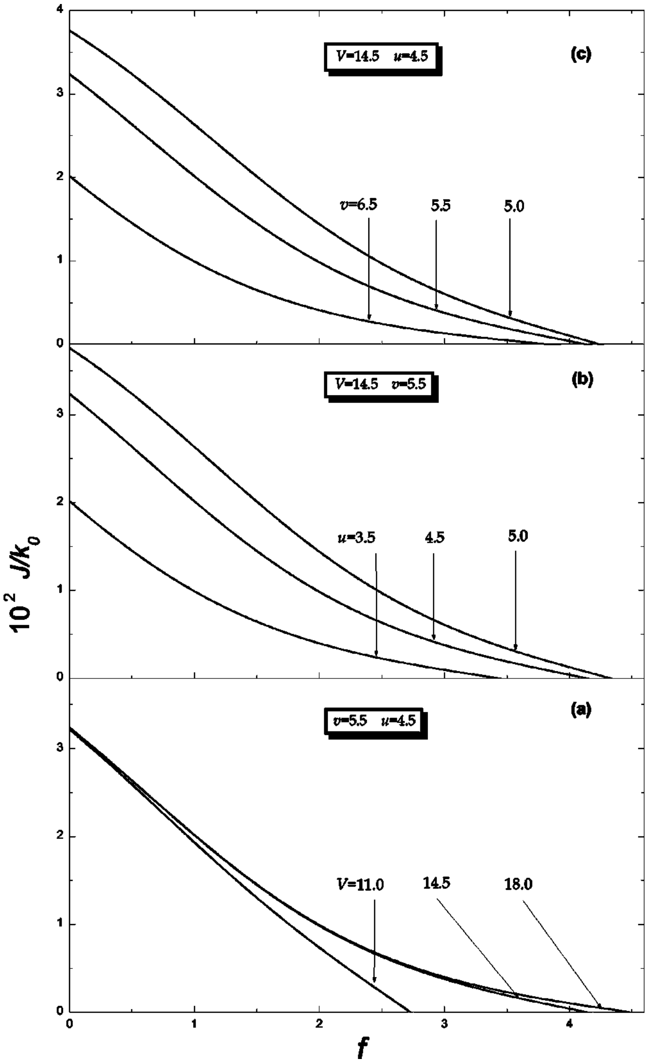

FIG. 2. Particle current as a function of the external load, Eq. (12), with one parameter of the potential varied [V for (a), u for (b), v for (c)] and the other two kept constant. For all curves $\gamma/k_0 = 0.1$.

$$\mathcal{U} = 2LJ. \tag{17}$$

With the set of parameter values taken from Ref. [21]: $V = 14.5$, $v = 5.5$, $u = 4.5$, $2L = 10^{-8}$ m, $k_0 = 10^3$ s^{-1}, and $\gamma = 10^2$ s^{-1}, the particle velocity $\mathcal{U}$ at zero force is 3.24×10^{-7} m/s and the stopping force F_s (at $T = 300$ K) is 6.9×10^{-12} N. It is worthy of mention that the calculated values of the velocity and the stopping force reproduce the corresponding experimental data for molecular motors [6] within an order of magnitude.

B. Efficiency

The model discussed in this paper is based on an obviously nonequilibrium mechanism. Every time the potential is rapidly changed, the particle distribution relaxes to the equilibrium profile and, as a result, some portion of the energy input into the system is dissipated to the thermal bath as a heat. The energetic efficiency is one of the simplest thermodynamical characteristics indicating to which degree potential fluctuations are converted into useful work.

As for any macroscopic motor, the energetic efficiency of a Brownian motor is defined as the ratio η of the power output P_{out} to the power input P_{in}

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}. \tag{18}$$

The output power is the mechanical work (per unit time) done against the external force, $P_{\text{out}} = F\mathcal{U}$. It is easily found using Eqs. (12) and (17). Following Refs. [12,24], the power input into the system stemming from the potential modulation can be written as

$$P_{\text{in}} = 2u[\gamma^+(\rho_2^+ - \rho_1^+) + \gamma^-(\rho_1^- - \rho_2^-)]. \tag{19}$$

Indeed, referring to Fig. 1, in each flip from the state $+(-)$ to $-(+)$ the particle gains or releases the energy $2u$, according to whether it is in well 2(1) or 1(2); on average, $\gamma^{+(-)}$ such flips occur in a unit time. With Eq. (8) for the occupancy probabilities, Eq. (19) reads

$$P_{\text{in}} = \frac{4u\gamma^*}{\Gamma + \Sigma}(\alpha_1^+ + \beta_1^+ - \alpha_2^+ - \beta_2^+). \tag{20}$$

Finally, rewriting the above relation for P_{in} in terms of the model parameters [in view of Eq. (7)] and substituting P_{in} and P_{out} into Eq. (18), we arrive at the desired expression for the efficiency of the Brownian motor. In the case of the optimal modulation ($\gamma = \gamma^+ = \gamma^-$), this expression takes the form

$$\eta(f) = \frac{f}{u}\,\frac{\sinh(u-f) - e^{-V+v}[\sinh(u+f) + 2\zeta\sinh(2f)]}{\sinh(u-f) + e^{-V+v}\sinh(u+f)}. \tag{21}$$

One can ascertain that $P_{\text{in}} > P_{\text{out}}$, i.e., $\eta < 1$, for all values of the parameters when $P_{\text{out}} > 0$, as it must be. The current and hence P_{out} monotonically diminish with f and become zero at $f = f_s$. Interestingly, the dependence $P_{\text{in}}(f)$ exhibits more complicated behavior. For large V, more precisely for $V > v + 2u$, the power input also drops with f and becomes zero at $f = f_0$. Note that the inequality $f_s < u < f_0$ holds, both f_s and f_0 tending to the same limit u (the former from the left, the latter from the right) at $V \to \infty$. Thus only at very large V, P_{in} and P_{out} change sign simultaneously and the energy transduction is completely reversible (a similar observation was made for the ECC model [23]). For $V \leqslant v + 2u$, the dependence $P_{\text{in}}(f)$ can exhibit nonmonotonic behavior and, what is especially important, the power input remains positive for all values of f. The latter circumstance implies that the process is not reversible when the highest potential peak is lower than $v + 2u$. This remark is also in agreement with the conclusion of Ref. [23], pointing to a formal analogy between the present model and the ECC model.

The efficiency increases with γ for an adiabatically slow modulation, $\gamma \ll k_0$, where the kinetic approach holds. Figure 3 illustrates the variation of the function $\eta(f)$, Eq. (21), with

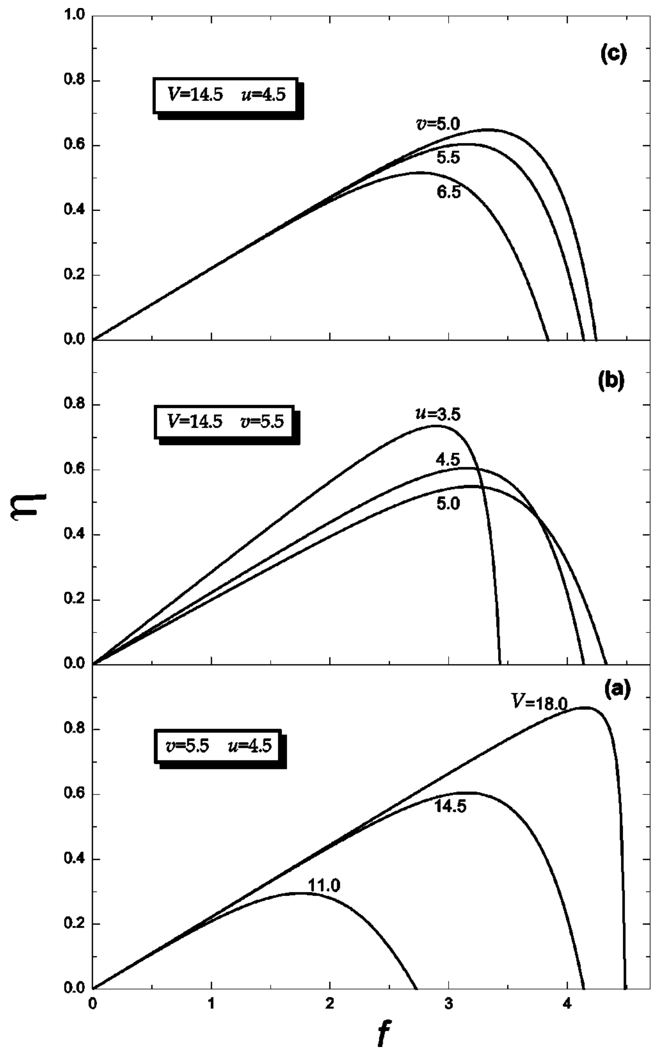

FIG. 3. Efficiency as a function of the external load, Eq. (21), with the parameters of the potential varied as in Fig. 2. For all curves $\gamma/k_0=0.1$.

the parameters of the potential. All the curves exhibit a similar nonmonotonic behavior: The efficiency is equal to zero at $f=0$, linearly increases with f in the region $f \ll f_s$, reaches its maximal value η_m, and rapidly goes to zero when f approaches f_s. An initial increase in efficiency with f is attributed to the equalization of the potential wells and hence to the lowering of the energy loss. When $f \to f_s$, P_{out} goes to zero whereas P_{in} remains finite, and hence $\eta \to 0$ (except the idealized case of $V=\infty$ when $\eta=1$ at $f=f_s$), which is in agreement with the conclusion of Refs. [2,14] that the efficiency vanishes at stalling conditions ($f=f_s$). The maximum of $\eta(f)$ arising from the competition between the two above-mentioned effects is located near f_s.

As Fig. 3 [panel (b)] shows, the lesser u, the higher the efficiency. In particular, a linear initial growth of η is characterized by its slope proportional to u^{-1}. The point is that a drop in the relative well energy u cuts the relaxation-induced energy loss and hence leads to an increase in the motor efficiency. It should be remembered, however, that as u lowers, the current and the stopping force also reduce (see Fig. 2), so that the operating f range of the ratchet shrinks.

There exists another more attractive way to achieve a high efficiency. As panel (a) of Fig. 3 suggests, the maximal efficiency (and the operating range) of the motor grows with rising V. Particularly striking are very high values of η at forces close to f_s for large V. In this range of f and provided $\varepsilon \ll 1$, Eq. (21) is reduced to

$$\eta \simeq \frac{f}{u}\left[1+\frac{1+\zeta}{f_s-f}u\varepsilon\right]^{-1}. \tag{22}$$

Using this approximate expression, one can readily see that the efficiency reaches its maximum

$$\eta_m \simeq 1-2\sqrt{(1+\zeta)\varepsilon} \tag{23}$$

at $f=f_m \simeq [1-\sqrt{(1+\zeta)\varepsilon}]u<f_s$. As expected, the corresponding current $J(f_m)/k_0 \simeq (1/2)\sqrt{\varepsilon/(1+\zeta)}ue^{-v}$ is very small but nonzero. It follows from Eq. (23) that the maximal efficiency of the motor tends to the ideal limit of unity, as the highest potential barrier increases. This conclusion is the main result of the paper. The idea behind it is easy to comprehend. At large values of V, the stopping force is close to its limiting value u (due to suppression of the backward motion). When the load force f is near f_m, $(u-f_m) \ll 1$, the difference between the well energies $2(u-f)$ becomes very small. This means that the motor works under quasiequilibrium conditions at every given time instant and the energy loss can be made arbitrarily small, in agreement with the observation made in Ref. [14] that the main energy loss results from backward steps.

Let us discuss the efficiency of the model in terms of the Fokker-Plank approach, restricting the consideration to the optimal modulation regime. The power output is easily found using Eqs. (15) and (17). The power input is determined by [12,24]

$$P_{\mathrm{in}}=\gamma\int_0^{2L}[U^+(x)-U^-(x)][\mathcal{P}^-(x)-\mathcal{P}^+(x)]dx. \tag{24}$$

The main contribution into the integral is attributed to the vicinity of the potential wells located at $L/2$ and $3L/2$, where the potential profiles are assumed identical, i.e., $U^+(x) \simeq U^-(x)+2u$ for x close to $L/2$ and $U^-(x) \simeq U^+(x)+2u$ for x close to $3L/2$. In the low-frequency domain, $\zeta \gg 1$, this leads to the power input estimated as $P_{\mathrm{in}} \simeq 4u\gamma R(L)$ and hence to the efficiency, Eq. (18), expressed by

$$\eta \simeq \frac{f}{u}\left[1+\frac{2J^+(0)}{\gamma R(L)}\right]. \tag{25}$$

The current $J^+(0)<0$ representing the backward motion vanishes when $V \to \infty$ and Eq. (25) coincides with the prediction of the kinetic approach, Eq. (23), in this limit.

Our last remark in this section concerns so-called *reversible* ratchets [24] which also achieve the maximal possible efficiency at certain conditions. Though the operating mechanism of this ratchet type is essentially different (potential barriers and wells are modulated out of phase and vary gradually, whereas our model implies in-phase rapid modulation), the result for the efficiency appears very similar to that obtained here [cf. Eq. (23) with Eq. (15) in Ref. [24]]. The origin of such likeness will be discussed elsewhere.

IV. CONCLUSIONS

In this paper we have considered a flashing ratchet model where a particle moves unidirectionally in a periodic potential (with two nonequivalent wells within a period) flipping between two identical states shifted by a half a period. Within the framework of the kinetic approach, we have obtained explicit expressions for the steady-state current, Eq. (12), and the energetic efficiency, Eq. (21), which are the quantities of paramount importance in the context of Brownian motor operation. Using these expressions, we have analyzed and discussed in detail the role of the model parameters in the particle transport. The main results of the kinetic consideration have been supported by the simple estimates obtained within the Fokker-Plank approach.

The appeal of the model is that its efficiency can be impressively high. Indeed, with the set of parameter values proposed in Ref. [21] (see the last paragraph in Sec. III A), the maximal efficiency η_m found here is 0.6, whereas η_m lesser than 0.05 was reported in Ref. [21]. The reason is that our model can be tuned so as to take the maximum advantage of the asymmetry between the rightward and leftward motion and to involve, in essence, no useless stages in the working cycle. Moreover, the efficiency of the Brownian motor considered here is shown to tend to the ideal limit of unity provided that the highest peaks of the potential are high enough to become factually impenetrable. In other words, such a high efficiency is reachable when the backward motion is locked and thermally activated hopping between energy wells occurs under quasiequilibrium conditions at every given time instant and the energy loss can be made arbitrarily small. We have thus demonstrated with the analytically treatable example that the scenario of the potential modulation proposed in Ref. [10] leads to high-efficiency flashing ratchets not only without (as shown earlier [14]) but also with diffusive steps involved in unidirectional motion generation.

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